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Estimates of Vertical Eddy Diffusion Due to Turbulent Layers in the Stratosphere

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FOR THE COMMANDER

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Chief Scientist

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Vertical transport in the stratosphere occurs by many processes and the question of the relative importance of stratified turbulence is considered in the context of a "vertical stack model" described in a previous report. The "vertical stack model" was designed to estimate the effect of intermittent turbulent layers (separated by essentially laminar flow) and it is here examined in some detail. In particular, a number of less obvious implications are discussed and a list of experimental tests of the model is suggested. It is concluded that the rate of turbulence for vertical transport in the stratosphere remains an open question.

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1. INTRODUCTION

The present concern about the vulnerability of stratospheric ozone to pollution raises the question of how material moves vertically in that stable environment. In the case of an homogeneous fluid (with neutral stability), turbulence plays a dominant role in mixing and vertical transport. The situation is markedly different in the case of a stably stratified fluid which, of course, is the case of interest in studies of the stratosphere. In this case, the turbulence confines itself to thin horizontal layers separated by non-turbulent layers. The turbulent layers, which are caused by the Kelvin-Helmholtz (KH) instability and which have the geometry (roughly) of a flat disc ("blini" or pancake), usually occupy a small fraction of the fluid volume (for example, 5 percent) and in the case of the stratosphere their vertical extent is presumably of order 200 meters. In contrast, their horizontal extent is typically in the range of a few tens of kilometers. The same sort of situation prevails in the upper ocean but there the vertical dimension is reduced to order 10 centimeters.

The vertical eddy diffusion coefficient, K_e, for the stratosphere over long time averages (based on radioactive fallout measurements) ranges from about 0.1 m^2/sec at the equator to 1.0 m^2/sec at the poles. This estimate (Junge),[1] of course,

(Received for publication 1 February 1979)

1. Junge, C.E. (1963) Air Chemistry and Radioactivity, Academic Press, New York, p 250.

includes not only the effects of the above-mentioned small scale turbulence, but it also includes global circulation and other effects as well. An important but unanswered question is whether or not the small scale turbulence plays a significant role in the overall value of K_e.

One method to estimate K_e due to turbulence depends upon estimates of the dissipation rate, ϵ, based in turn upon the spectral analysis of in situ velocity fluctuations. Lilly[2] has done this and found $K_e \sim 0.01\ m^2/sec$ which is very small. The purpose of the present report is to closely examine a newer method to estimate K_e (due to small scale turbulence) which is based on measurements of the vertical profile of the horizontal mean winds made in conjunction with measurements of the vertical temperature profile. Such wind measurements have been made in our field experiments by means of rocket trails and by others (Van Zandt et al)[3] using radar techniques. Temperature profiles are obtained from measurements from free balloons carried out simultaneously. Rosenberg and Dewan[4] were the first to describe this technique and the reader will find in that report an extensive review of the literature leading to the main concepts involved.

More precisely, the goal of this report is to make explicit all the assumptions behind our model and to examine them in detail. Also, generalizations will be described which will show when it is permitted to use the relation employed by Lilly[2] (and in the context of the ocean by Woods and Wiley[5]) given by

$$K_e = P^* K_L \tag{1}$$

where P^* is the fraction of the vertical dimension which is occupied by turbulent flow and K_L is the effective average eddy diffusion within the turbulent layers. It will be shown that when K_L is large, Eq. (1) is not valid and one must use

$$K_e = \frac{P^* \overline{L^2}}{2 \Delta t_f} \tag{2}$$

where L is related to turbulent layer thickness and Δt_f is the average time between changes of the vertical Richardson number profile. Eqs. (1) and (2) will be derived

2. Lilly, W., Waco, D., and Adelfang, S. (1975) Stratospheric mixing estimated from high-altitude turbulence measurements by using energy budget techniques. The Natural Stratosphere of 1974, CIAP MONOGRAPH 1, DOT-TST-75-51 pp 6-81 to 6-90.

3. Van Zandt, T.E., Green, J.L., Gage, K.S., and Clark, W.L. (1978) Vertical profiles of refractivity turbulence structure constant, Radio Sci. 13:819-829.

4. Rosenberg, N.W., and Dewan, E.M. (1975) Stratosphere Turbulence and Vertical Effective Diffusion Coefficients, AFCRL-TR-75-0519, ERP No. 535.

5. Woods, J.D., and Wiley, R.L. (1972) Billow turbulence and ocean microstructure, Deep Sea Research and Oceanic Abst. 19:87-121.

from two approximations resulting from the vertical stack model which was first described in Rosenberg and Dewan.[4]

In addition to discussing certain subtleties of the vertical stack model as well as some details of data analysis, this report compares the behavior of turbulent layers in the atmosphere to those in the upper ocean. The latter is of interest because the upper ocean is much more accessible than the stratosphere and acts, to some extent, as a "laboratory model" of it. Finally, a list of important experimental questions raised by the model is given. Preliminary results using the present approach were given in Rosenberg and Dewan[4] hereafter designated by I; and an estimate of $K_e \sim 0.3\ m^2/sec$ was given. If such an estimate were to be validated by future measurements, then turbulence would be important in vertical transport in the stratosphere. At present, however, the question remains open.

2. REMARKS ON THE SIMILARITIES AND DIFFERENCES IN THE TURBULENCE OF THE UPPER OCEAN AND STRATOSPHERE

The studies by Woods[6] and Woods and Wiley[5] of the thermocline and layered turbulence to be found there are, as has been mentioned, very relavant to stratospheric research (cf. I). The thermocline, or region where there is a stable temperature gradient near the ocean surface, seems to be divided into regions which are called "sheets" across which are very sharp density gradients, separated by "layers" where the gradients are much smaller. Turbulence occurs in the vicinity of the sheets whereas the layers can be considered to be effectively laminar.

An important role seems to be played in thermocline turbulence by "buoyancy waves," sometimes called internal waves. These waves (as was shown theoretically by Phillips)[7] can enhance local mean shears in a manner which results in lower R_i. When $R_i < 0.25$, turbulent breakdown can take place, and when turbulence commences, as described by Woods and Wiley,[5] an hysteresis phenomenon is set into motion. The turbulence causes the layer to thicken, but this thickening causes R_i to increase. Turbulent entrainment is the cause of the thickening and the latter can, on the basis of energy considerations, continue until $R_i = 1$. Once $R_i = 1$, however, the turbulence can no longer extract energy from the mean shear and hence $R_i = 1$ represents the "cut off." Thus, during the "active life" of the turbulence, the layer expands, and at $R_i = 1$ motion decays and this decay endures over a period of time. In the case of the ocean, the layer expands by a factor of four (during the active life). This was determined by observation by Woods and Wiley[5]

6. Woods, J. D. (1968) Wave-induced shear instability in the summer thermocline, J. Fluid Mech. 32:791-800.

7. Phillips, O. M. (1969) The Dynamics of the Upper Ocean (Cambridge Univ. Press).

who also estimated it on the basis of a simple model which assumes that the horizontal velocity difference across the layer, ΔV, as well as the temperature difference, $\Delta \Theta$, remain constant during expansion. After layer expansion, a "collapse" takes place. This presumably is due to the effect of incomplete mixing followed by the return of fluid parcels to their stable depths (Koop).[8]

The appearance of tropospheric clear air turbulence (CAT) on radar has a striking similarity to that seen in the ocean and laboratory. But there are also important differences. For example, in the case of Browning and Watkins' observation,[9] there is a complete absence of collapse subsequent to the expansion. The expansion effects themselves don't seem to be visible on the radar. In addition, as will be shown below, one would expect that in the case of the atmosphere there is much less expansion than in the ocean, and from the above remarks, it should be clear that this in turn would help explain the lack of collapse. Specifically, with less turbulent entrainment, there would be less fluid to mix and thus mixing could be more thorough, and hence there would be far more "homogenization" and less migration of parcels to their stable altitude.

In order to explain the hypothetical lack of turbulent layer spread in the atmosphere, we note that the above-mentioned assumption of constant $\Delta\theta$ over time across the layer made in the case of the ocean is probably invalid for the case of atmospheric turbulence. In the case of the thermocline the temperature profile is determined by physical mixing of water (Phillips)[7] and therefore, between mixing events it remains unchanged. In contrast, the θ profile for air (here we identify θ with potential temperature) is due predominantly to radiative effects and between the mixing events the "steps" in the profile would tend to become smoothed out. It is, therefore, more appropriate to replace the constant $\Delta\theta$ condition with a constant $d\theta/dz$ or vertical gradient (or buoyancy frequency, N_B) condition. When this is taken into account and when we furthermore replace the $R_i = 1$ cut off value with the one advocated by Garrett and Munk[10] (which was due to Thorpe) namely $R_i = 0.4$, then it can be shown (see I) that the expansion factor is a mere 1.26 in contrast to 4 ($R_i = 1$ would have given an expansion factor of 2).

Another point raised by Woods and Wiley[5] is that "sheet-ensembles" might be generated by repeated billow events. This in turn raises the possibility that the same sort of thing occurs in the stratosphere.

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8. Koop, C.G. (1976) Instability and Turbulence in a Stratified Shear Layer, USCAE 134, Univ. of Southern California, School of Engineering, Dept. of Aerospace Engineering.

9. Browning, K.A. (1971) Structure of the atmosphere in the vicinity of large amplitude Kelvin-Helmholtz billows, Roy. Met. Soc. Quart. J. 97:283-299.

10. Garrett, C., and Munk, W. (1972) Oceanic mixing by breaking internal waves, Deep Sea Research 19:823-832.

3. THE VERTICAL STACK MODEL FOR K_e

3.1 Assumptions

First, we shall assume that there is no vertical transport taking place between the turbulent layers. Certainly a significant amount of transport by molecular diffusion (as compared to turbulent diffusion) can be ruled out on the basis that the molecular diffusion is of order 10^{-4} m^2/sec in the lower stratosphere whereas K_e (according to Lilly)[2] is 10^{-2} m^2/sec, or (according to I) 10^{-1} m^2/second. The second assumption we make is that turbulence occurs randomly with altitude and time. With no information to the contrary, this is a valid a priori assumption. Next we assume that, before the turbulence decays in the layer, total mixing takes place. This assumption receives support from two entirely independent pieces of experimental information. The first is described in the report by Mantis and Pepin[11] where it was shown that in the stratosphere there are layers where the temperature gradient with respect to altitude is nearly equal to the adiabatic lapse rate. This would, of course, be consistent with the idea that total mixing occurs because such mixing would presumably leave behind an adiabatic lapse rate. The second piece of evidence is from Browning and Watkins[9] and also from Atlas et al.[12] These authors looked at turbulent layers ("CAT") in the troposphere with radar and they found that after the turbulent activity had presumably decayed (following the instability) there remained two widely separated layers of very strong reflection each located at an edge of the original layer. This indicates a strong gradient of temperature at each edge which is precisely what would be found if essentially total mixing occurred within the layer. In a sequel to the present report, I shall give a theoretical argument which gives still further justification to the total mixing assumption. (The case of small mixing will also be examined below as already mentioned.)

Finally, it is assumed that the horizontal rearrangement of the layer and transported material will have little effect on vertical transport. This last assumption is not unusual in oceanography, and it permits the use of a simple one-dimensional model (see Garrett and Munk).[10]

3.2 The Model for K_e

The classical eddy diffusion concept was formulated for the case where turbulence is relatively uniform. That is to say, it would not apply to our case where

11. Mantis, M.T., and Pepin, T.J. (1971) Vertical temperature structure of the free atmosphere at mesoscale, J. Geophys. Res. 20:8621-8628.

12. Atlas, D., Metcalf, J., Richter, J., and Gossard, E. (1970) The birth of "CAT" and microscale turbulence, J. of the Atm. Sci. 27:903-913.

certain regions of the fluid are turbulent while others are laminar. The usual approach (see for example, Pasquill)[13] is briefly as follows.

Let C be the concentration of material as a function of position and consider the vertical flux, Q, due to the turbulence. Letting w' be the fluctuation of vertical velocity and C' the fluctuation in concentration, then $Q = \overline{w' C'}$ where the overbar signifies a suitable average. C' is given by $C' = -\ell \partial \overline{C}/\partial Z$ where ℓ was originally called a "mixing length" but is now considered to be a characteristic spatial dimension of the turbulence. K_e is then defined by analogy with Fourier's heat transfer equation

$$Q = -K_e \, \partial \overline{C}/\partial Z \tag{3}$$

and thus $K_e = \overline{w'\ell}$.

This approach fails for the case we are considering, therefore we re-examine it in detail in order to arrive at an extension of the eddy diffusion concept which can be of use in stratified fluids.

In Figure 1 the fluid is modeled by a medium located between a source at the top and a sink at the bottom. Within this medium are few randomly spaced mixing layers and we assume that the initial concentration profile is linear as indicated in Figure 1a. Figure 1b indicates the effect upon the profile of the mixing layers where the assumption of total mixing accounts for the vanishing gradients within the layers. Note that there results a net downward transport from this mixing. This fact is essential for the understanding of this process of vertical transport. Note also that if the layers were to be locked to this initial configuration, no further vertical transport through the medium would be possible. We shall assume that whenever a layer is in contact with either the source or the sink, the mixing action will convert its concentration to that of the source or to zero in the case of the sink. In the context of stratospheric transport the "instantaneous sink" is justified from the fact that the residence time is of the order of years for the stratosphere, whereas in the troposphere it is given in terms of weeks. Storm systems, convection, and other transport mechanisms operate in the latter and these are responsible for the great disparity in residence times.

Next we imagine that the medium in Figure 1 is subjected to a succession of profiles of randomly spaced mixing layers of variable thickness. Figure 2 shows a computer simulation of this process after a significant number of profiles. As can be seen, the C (Z) profile departs from the original linear shape and becomes very choppy. Thus this form of transport is very "lumpy" and the "diffusion" represented by the K_e derived below must be viewed in this context.

13. Pasquill, F. (1962) Atmospheric Diffusion, Van Nostrand Co. Ltd.

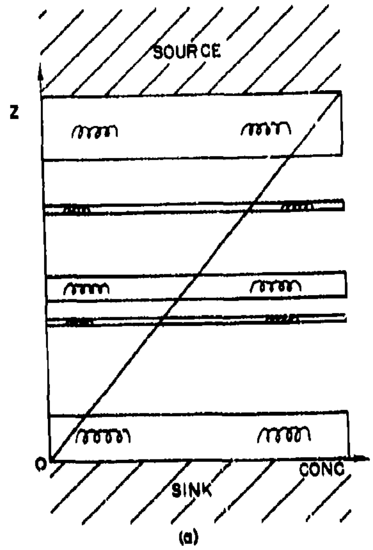

Figure 1a. Vertical Stack Model of Turbulent Mixing Layers. Ordinate is altitude, slanted line is concentration profile and abscissa designates amount of concentration

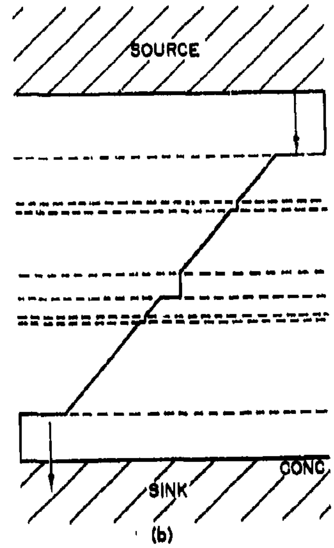

Figure 1b. Effect of Mixing Layers on Concentration Profile. Total mixing inside layers is assumed. Net downward transport from source to sink is shown

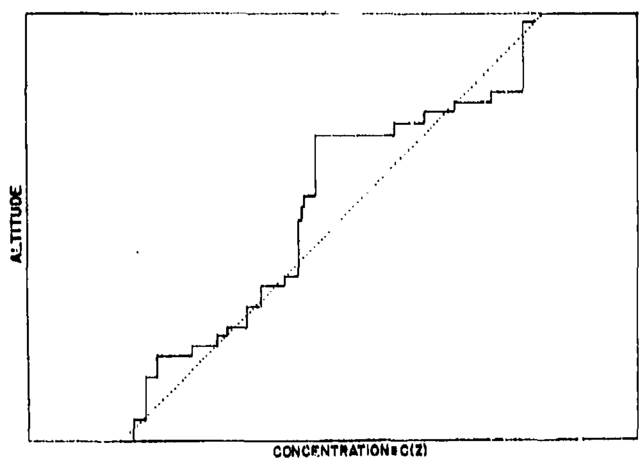

Figure 2. Computer Simulation of the Effect Upon Linear Concentration Profile (dotted line) by a Series of "Time Frames" of Turbulent Mixing Layer Profiles. In this simulation, statistics of thickness and occurrence are based on experimental wind profiles in the stratosphere. Discrete nature and step like structure is illustrated

Presumably, in the actual stratosphere, the mixing layers occur at random altitudes, with random thickness at random times. To model this in a simple manner we conceptualize the process as occurring in discrete, evenly spaced time frames with a single profile of mixing layers in a given frame. The duration of these time frames we denote by Δt_f.

From Eq. (3) we define

$$K_e = -Q/(\partial \overline{C}/\partial Z) \ . \tag{4}$$

We must now determine the value of the flux Q. This flux is extremely sporadic in the sense that no material is "dumped" out of the bottom of the stack (Figure 1) until a layer forms there. We thus envision the process in a manner which regards it only over a long period of time and in an averaged out sense. On average, the stack is in "steady-state" with as much material entering the top as leaving the bottom. The profile C (Z) is, on average, a roughly linear profile.

To calculate Q we define Δt_b as the average time between the occurrences of layers at the bottom of the stack (in contact with the sink). This can be related to Δt_f from the definition of P*. Thus

$$P^* = \Delta t_f/\Delta t_b \ . \tag{5}$$

Since P* is very small (for example, 0.05), Δt_b in general will be much larger than Δt_f. Next we define the symbol Δn to equal the amount (on average) of the material in the bottom layer which is then "dumped out" into the sink due to mixing. Let L in Figure 3 represent the average thickness of the bottom layer. Assuming the linearity of C (Z) the average concentration in the layer is $1/2 \cdot L \cdot \partial C/\partial Z$ and from this one obtains

$$\Delta n = (A\ L) \cdot (L/2)\ \partial\ \overline{C}/\partial Z \tag{6}$$

where A is the area of bottom surface and A L is the volume of matter. Q therefore is now obtained from Eq. (6) as

$$Q = \frac{(-\Delta n)}{A\,\Delta t_b} = -\left(\frac{L^2}{2}\ \frac{\partial\,\overline{C}/\partial Z}{\Delta t_b}\right) \tag{7}$$

(the minus occurring because the flow is downwards).

Therefore, from Eq. (4)

$$K_e = \frac{1}{2}\frac{\overline{L^2}}{\Delta t_b} \ . \tag{8}$$

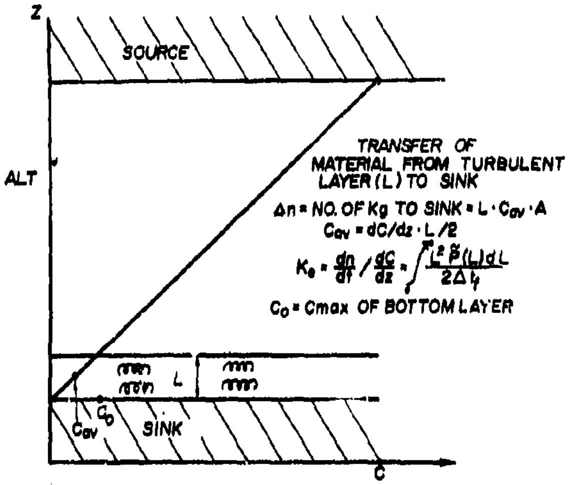

Figure 3. Diagram for K_e Vertical Stack Model Relating Wind Profile Statistics, $\bar{P}$ (L) to Vertical Transport. Randomness and stationarity relate the dumping effects of the bottom layer to the entire stack structure

As mentioned, the value of $\overline{L^2}$ must relate to data by means of some sort of average; however, before we see how this is carried out in practice it is important to understand the subtle meaning of L. For example, it is definitely not to be understood as the average thickness of a turbulent layer in the fluid. Instead it is the average thickness of a turbulent layer which extends above a previously given altitude.* Recall that the bottom of the stack is at a given fixed location. The next section will reflect this consideration by the way in which the data is processed.

3.3 $\overline{L^2}$ and Analysis of Data

We now consider the experimental determination of $\overline{L^2}$ to be used in Eq. (8) to obtain K_e. The primary data consists of vertical profiles of the mean horizontal winds. In addition one also needs the temperature profiles. The data are combined to give the Richardson number (R_i) profiles where

$$R_i = \frac{(g\, d\Theta / dZ)}{\bar{\Theta}} \Bigg/ \left[\frac{d\bar{u}}{dZ}\right]^2 \tag{9}$$

and g is the acceleration of gravity, Θ the potential temperature, $\bar{\Theta}$ its average, and $\bar{u}$ is the average horizontal wind velocity. In order to determine the potential existence of turbulence, the usual criterion of $R_i < 0.25$ is used (cf. Ref. 1).

The purpose of the data analysis is to search the wind profiles for supercritical shears ($R_i < 0.25$). To illustrate the technique we describe in detail what was done

*Thus on average the layer thickness is roughly twice the average value of L.

to obtain the results given in Ref. I. In that case, data were available at 25 m altitude intervals. Since no temperature data were available, we used a standard atmosphere and set the numerator of Eq. (9) to that (constant) value. A computer program searched the wind profiles for unstable zones as follows. For a given profile the search began at the bottom point. The shear between this point and the one a step higher was calculated. If $R_i > 0.25$, the sample was counted as non-critical (non-turbulent) and the program considered the second point up as its initial altitude and repeated the measurement as before. When an altitude point was reached where the shear between that altitude and the subsequent one was super-critical, the program would then calculate the shear between the initial altitude and one two steps higher (50 m), then three steps (75 m) and on until an altitude span was reached where $R_i > 0.25$. The program would then return downwards one step and record the altitude difference as a value of L. When all these values were obtained they were counted up in the form of a cumulative fraction, $P_I(L)$.

$P_I(L)$ is defined as the fraction of altitude samples which had a supercritical shear between the given altitude and one a total distance L above it (or greater). In other words

$$P_I(L) = \sum_{L}^{L\,max} \Delta P(L) \tag{10}$$

where $\Delta P(L)$ represents the fraction of samples with L between the value of L and $L + \Delta L$, where ΔL = the altitude interval between samples (for example, 25 m). From Eq. (10)

$$P_I(L) = \sum_{L\,min}^{L\,max} \Delta P(L) - \sum_{L\,min}^{L-\Delta L} \Delta P(L) \tag{11}$$

where $L\,min = \Delta L$ and $L\,max$ is the maximum span of super-critical shear. Recalling that P^* is the average fraction of vertical profiles that are occupied by unstable shears (potentially turbulent) we have

$$P^* = \sum_{L\,min}^{L\,max} \Delta P(L)\,. \tag{12}$$

From Eqs. (11) and (12)

$$P_I(L) = P^* - \sum_{L\,min}^{L-\Delta L} \Delta P(L)\,. \tag{13}$$

(L can be replaced by $m\Delta L$ where m is an integer from 1 to some upper limit.) $\Delta P(L)$ is then calculated from $P_I(L)$ by

$$\Delta P(m\Delta L) = P_I(m\Delta L) - P_I[(m+1)\Delta L] \tag{14}$$

as can be seen from Eq. (13).

Next, one needs a probability in order to calculate $\overline{L^2}$ and for this we define

$$\tilde{P}(L) = \frac{\Delta P(L)}{P^*} \tag{15}$$

which represents the fraction of potentially turbulent zones found which have the span L. Finally

$$\overline{L^2} = \sum_{m=1}^{m=max} (m\Delta L)^2 \tilde{P}(m\Delta L). \tag{16}$$

K_e is then determined from Eq. (8) or, using Eq. (5)

$$K_e = \frac{P^* \overline{L^2}}{2\Delta t_f}. \tag{17}$$

Ref. I carried out these procedures. Figure 4 shows $\tilde{P}(L)$ based on 10,000 samples of wind profile data published by Miller, Henry et al[14] at NASA. To summarize the findings of Ref. I, we used Eq. (17) to obtain $\overline{L^2} = 1.2 \times 10^4\ m^2$, Eq. (12) to obtain $P^* = 4.8 \times 10^{-2}$, and Eq. (17) then gave $K_e \sim 0.19\ m^2/sec$; (Δt_f = 1500 sec was used and this will be discussed at length below). The previously discussed expansion factor (due to layer spread) results in a factor 1.6 increase in K_e and thus we arrived at the estimate of $K_e \sim 0.3\ m^2/second$.

3.4 Extension to the Case of Small Mixing

Having obtained Eq. (16) on the assumption that complete mixing takes place within a turbulent layer we now turn to the opposite case where there is only a small amount of mixing and derive Eq. (1) which was used by Lilly et al,[2] from the vertical stack model. K_L is defined as the eddy diffusion coefficient within a mixing layer. Turning again to Figure 3 and the bottom layer of the stack, we estimate Δn in the new context. Let Δt_m represent the duration of the mixing and let K_L be regarded as constant during this time and zero afterwards. (This simplification will be made less artificial below by considering the case of K_L monotonically decreasing in time without limit for actual mixing time.) Let n_o be the amount of material inside the layer located over a unit horizontal area at t = 0 and let $n_{\Delta t_M}$ be this quantity at $t = \Delta t_M$. Then,

14. Miller, R. W., Henry, R. M., and Rowe, M. C. (1965) Wind Velocity Profiles Measured by the Smoke-Trail Method at Wallops Island, Virginia (1959-1962) NASA TN D-2037, see also NASA TN D-4365 (1968).

$$\frac{(\Delta n)}{A} = (n_0 - n_{\Delta t_M}). \tag{18}$$

Thus, to estimate Δn one must know n as a function of time as it diffuses out of the layer n. The definition of n is the amount of material inside the layer at time t over a unit horizontal area. As seen in Figure 3, C_0 is the maximum concentration of the layer thus

$$C(Z) = C_0 \frac{Z}{L}. \tag{19}$$

The heat equation which can be derived from Eq. (4) with K_e replaced by K_L is

$$\frac{\partial C}{\partial t} = K_L \frac{\partial^2 C}{\partial Z^2}. \tag{20}$$

To complete the boundary value problem, it is assumed that the top of the turbulent layer is "non-conducting" that is, the normal derivative of C with respect to Z at the top surface will be zero (that is, at Z = L) for all times after t = 0, and that at Z = 0, C is held to the value zero (being in direct contact with the sink).

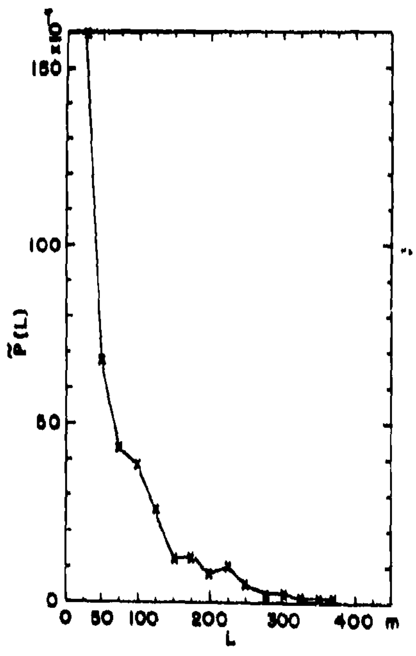

Figure 4. Histogram for Normalized Distribution, $\tilde{F}(L)$

This sort of problem is well known in heat conduction, and the solution is obtained in the classic manner of separation of variables and Fourier series expansion (see for example, Ingersoll et al, p. 126).[15] One thus arrives at

$$C(Z, \Delta t_M) = C_o \sum_{m=1}^{\infty} \exp\left\{-\left(\frac{m\,\Pi}{\ell}\right)^2 K_L \Delta t_M\right\} \sin\left(\frac{m\,\Pi Z}{\ell}\right) \cdot b_m \tag{21}$$

where b_m is the Fourier coefficient given by

$$b_m = \frac{2}{\ell}\left[\int_o^{\ell} \frac{2Z \sin aZ}{\ell}\, dZ + \int_{\ell/2}^{\ell} \left[\frac{2-2Z}{\ell}\right] \sin aZ\, dZ\right] \tag{22}$$

where $\ell = 2L$ and $a = (m\,\Pi/\ell)$. Thus

$$b_m = \frac{4}{(m\,\Pi)^2}\; 2\sin\left(\frac{m\Pi}{2}\right) - \sin(m\,\Pi). \tag{23}$$

The concentration profile as a function of time is then

$$C\,(Z\text{-}t) = C_o \sum_{m-1}^{\infty} \exp\left\{-\left(\frac{m\,\Pi}{\ell}\right)^2 K_L \Delta t_m\right\} \frac{4}{(m\,\Pi)^2}$$

$$\left[2\sin\left(\frac{m\,\Pi}{2}\right) - \sin(m\Pi)\right] \sin\left(\frac{m\,\Pi z}{\ell}\right). \tag{24}$$

The value of n (t) can be obtained from

$$n(t) = \int_o^{\ell/2} C\,(Z,t)\, dZ\,. \tag{25}$$

Thus

$$n(t) = \frac{4\,C_o\,\ell}{\Pi^3} \sum_{m-1}^{\infty} \left(\frac{1}{m^3}\right)\left(1-\cos\frac{m\Pi}{2}\right)\left(2\sin\frac{m\Pi}{2} - \sin m\Pi\right) \cdot$$

$$\exp\left[-\left(\frac{m\Pi}{\ell}\right)^2 K_L \Delta t_M\right] \tag{26}$$

and at t = 0

15. Ingersoll, L. R., Zobel, O. J., and Ingersoll, A. C. (1954) Heat Conduction, University of Wisconsin Press (Madison, Wisc.).

$$n_o = \frac{C_o \ell}{4} \tag{27}$$

(cf. Eq. (7) where $C_o = (\frac{\partial C}{\partial Z} \cdot L)$ and $\ell = 2L$).

Inserting Eqs. (27) and (26) into Eq. (18) and using $t = \Delta t_M$

$$\frac{\Delta n}{A} = \frac{C_o \ell}{4} \left[1 - \frac{16}{\Pi^3} \sum_{m=1}^{\infty} \frac{1}{m^3} \exp \left\{ - \left(\frac{m \Pi}{\ell} \right)^2 K_L \Delta t_M \right\} g(m) \right] \tag{28}$$

where

$$g(m) = \left[2 \sin \left(\frac{m \Pi}{2} \right) - \sin (m \Pi) \right] \left[1 - \cos \left(\frac{m \Pi}{2} \right) \right] . \tag{29}$$

This function, $g(m)$, takes on the values 2, 0, -2, 0, and so on, as m takes on the values 1, 2, 3, 4, etc.

To obtain K_e in the case of arbitrary mixing, we insert Eq. (28) into Eq. (7) and replace dC/dZ by C_o/L in Eq. (4):

$$K_e = \frac{L^2}{2 \Delta t_b} \left[1 - \frac{16}{\Pi^3} \sum_{m=1}^{\infty} \frac{1}{m^3} \exp \left\{ - \left(\frac{m \Pi}{\ell} \right)^2 K_L \Delta t_m \right\} g(m) \right] . \tag{30}$$

We first specialize this most general form to the large mixing case again by taking

$$K_L \Delta t_M >> \left(\frac{\ell}{m \Pi} \right)^2 \tag{31}$$

Under this condition, Eq. (30) becomes Eq. (17) [using Eq. (3)] again, thus there is consistency. The case of interest now is the small mixing condition of

$$K_L \Delta t_M << \left(\frac{\ell}{m \Pi} \right)^2 . \tag{32}$$

It can easily be shown that all the non-zero terms above the first in the sum given in Eq. (30) are negligible in this problem. Also $(32/\Pi^3) \approx 1.0$. Thus Eq. (30) can always be replaced by

$$K_e = \frac{L^2}{2\Delta t_b} \left[1 - \exp \left\{ - \left(\frac{\Pi}{\ell} \right)^2 K_L \Delta t_M \right\} \right] \tag{33}$$

for all practical purposes. Now, setting m = 1 in Eq. (32) allows us to expand the exponent

$$\exp\left\{-\left(\frac{\pi}{\ell}\right)^2 K_L \Delta t_M\right\} \approx \left(1-\left(\frac{\pi}{\ell}\right)^2 K_L \Delta t_M\right) \tag{34}$$

when Eq. (34) is valid, Eq. (33) leads to

$$K_e = \frac{\pi^2}{8} K_L \frac{\Delta t_M}{\Delta t_b} \tag{35}$$

and from Eq. (5)

$$K_e = (1.23)\, K_L P^* \frac{\Delta t_M}{\Delta t_f}\,. \tag{36}$$

The fact that

$$\frac{1}{2} \sum_{m=1}^{\infty} \frac{g(m)}{m} = \frac{\pi}{4} \tag{37}$$

can be used to show that the constant is actually closer to unity,* and we shall replace 1.23 by 1.
Thus

$$K_e = K_L P^* \tag{38}$$

which is the same as Eq. (1), provided that Eq. (32) holds, plus the added assumption that

$$\Delta t_M = \Delta t_f\,. \tag{39}$$

Recall that Δt_M represents a "mixing time" and Δt_f a growth time. Eq. (39) raises two questions: "What physical reason can be given for this equality; and what is implied when the equality does not hold?" These will be answered in the next section.

Finally, we ask what numerical values of K_L are consistent with Eq. (32) in the context of the stratosphere? For convenience we set as criterion

$$\frac{\pi^2 K_L \Delta t_M}{4L^2} = \frac{2}{3}\,. \tag{40}$$

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*This is done by retaining the approximation [Eq. (34)] but using the infinite sum instead of only the first term.

This we take for the maximum permissable value of K_L. Using the $L^2 \sim 1.21 \times 10^4\ m^2$, $\Delta t_M \sim 1,500$ sec (say) we arrive at

$$K_L \leq 2.18\ m^2/sec \tag{41}$$

for Eq. (32). The values reported by Lilly et al[2] are

$$0.26 < K_L < 1.0\ m^2/sec\ .$$

Thus his results are, in this sense, self-consistent. On the other hand, the values reported by Zimmerman et al.[16] namely $40 < K_L < 380\ m^2/sec$, violate Eq. (41). The issue seems not to be settled, especially if measured values of K_L are not known to be the "initial values" in all cases.

3.5 Growth Times, Decay Times, and Vertical vs Horizontal "Probability of Turbulence"

The "growth time" Δt_f is related to the exponential time constant for the instability, τ_g. The latter is given in the form of the following relation by Woods[6]

$$\tau_g = \frac{1.3\ (\text{shear})^{-1}}{0.27 - R_i} \tag{42}$$

which is valid for $0.05 < R_i < 0.20$. The crucial parameters are R_i and shear.

In contrast, the decay time could be estimated from

$$\Delta t_M = 3\ L \left(\frac{1}{u_f} - \frac{1}{u_i}\right) \tag{43}$$

where L is now assumed to be the outer scale of the turbulence, and u_i and u_f are the initial and final fluctuation velocities. Eq. (43) has been derived from

$$\frac{d\,(3/2\ u^2)}{dt} = -\epsilon \tag{44}$$

where $(3/2\ u^2)$ is the kinetic energy of the fluctuations (u here is to be understood as the root mean square velocity) and ϵ is the dissipation rate taken equal to u^3/L. The outer scale L is assumed constant in this calculation.

In order to compare Δt_M with τ_g, we must first have values for u_i and u_f. Let us choose $u_f = e^{-1}\ u_i$. Thus the "e-folding" mixing time, Δt_{M_e} is

16. Zimmerman, S.P., and Loving, N. (1975) Turbulent dissipation and diffusivities in the stratosphere using Richardson's technique. The Natural Stratosphere of 1974, CIAP MONOGRAPH 1, Final Report, DOT-DIAP, DOT-TST-75-51.

$$\Delta t_{M_e} = 3\, L \frac{(e - 1)}{u_i} = \frac{5.2\, L}{u_i} . \tag{45}$$

We now alter the form of τ_g so that it can be compared to Eq. (45). To do this we shall replace the shear in Eq. (42) with a relation involving u_i and use an appropriate value for R_i. In a future report concerning the degree of mixing subsequent to K-H breakdown it will be shown on the basis of a simple energy argument that

$$u_i = SL \left(\frac{1 - R_i}{3} \right)^{1/2} \tag{46}$$

where S is the shear across the layer of thickness L.

To choose R_i we turn to the observations of Browning.[9] In view of the fact that L is often less than his 200 m vertical distance resolution and in view of his range of R_i just before the billow event on Radar (0.15 - 0.3) we arbitrarily set $R_i = 0.15$ as being a "typical" value. Eq. (46) can be used to obtain S as a function of u_i and L.

$$S = \frac{u_i}{L} \sqrt{\frac{3}{1 - R_i}} = 1.88 \frac{u_i}{L} . \tag{47}$$

Putting this into Eq. (42) and again using $R_i = 0.15$ we obtain

$$\tau_g = \frac{L}{u_i} (5.76) \tag{48}$$

which should be compared to Eq. (45). Thus it is not unreasonable to expect

$$\Delta t_{M_e} \sim \tau_g \tag{49}$$

or

$$\Delta t_M \sim \Delta t_f . \tag{50}$$

In other words, despite a certain degree of arbitrariness in the choice of R_i, and so on, Eq. (50) is certainly not unreasonable.

As a further check on overall self-consistency, let us compare the Δt_f obtained somewhat experimentally in I with the following estimate of Δt_M ($\Delta t_f \sim 1500$ sec). Up to this point, Δt_M was used together with a fixed value of K_L in our calculations. We now let K_L be variable.

In this case, a good "definition" for Δt_M would be

$$K_L^o \, \Delta t_M \equiv \int_0^\infty K_L \, dt$$

where K_L^o is the value of K_L <u>at the beginning</u> of the turbulence. Here K_L will depend on the velocity fluctuations and we set

$$K_L(t) \equiv u(t)\, L \tag{52}$$

using

$$t = 3\,L\left(\frac{1}{u(t)} - \frac{1}{u_i}\right) \tag{53}$$

which is derived in the same manner as Eq. (43). One can solve for u (t) to obtain

$$u_I(t) = \frac{3\,L\,u_i}{3L + u_i t} \tag{54}$$

where u_I is the velocity in the first stage of decay.

As pointed out in Tennekes and Lumley[17] however, $\epsilon = u^3/L$ is no longer appropriate after the Reynold's number decays below about 10. At that point,

$$\epsilon = \frac{C'\,\nu\,u^2}{L^2} \tag{55}$$

becomes more appropriate. To match these two expressions for ϵ at $R_e = 10$ we must set $C' = 10$. Thus, for $R_e \leq 10$

$$\frac{d\,(3/2\;u^2)}{dt} = -\frac{10\nu\,u^2}{L^2} \tag{56}$$

which leads to

$$u_{II}(t) \equiv \tilde{u}\; e^{\frac{-10\nu}{L^2}\,\cdot\,\frac{1}{3}(t-\tilde{t})} \tag{57}$$

where u_{II} is the velocity in the last stage of decay and where $\tilde{u}$ is the transition velocity determined from

$$\tilde{u} = \frac{(R_e)\,\nu}{L} = \frac{10\,\nu}{L}\,. \tag{58}$$

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17. Tennekes, H., and Lumley, J. L. (1972) <u>A First Course in Turbulence</u> (The MIT Press, Cambridge, MA).

The time t for the transition to take place can be determined from Eq. (53) where u_f is replaced by $\tilde{u}$ and u_i determined from Eq. (46)

$$\tilde{t} = 3\,L\left(\frac{1}{\tilde{u}} - \frac{1}{u_i}\right). \qquad (59)$$

We can now estimate $K_L^o \Delta t_M$ from

$$K_L^o \Delta t_M = L \int_0^{\tilde{t}} u_I(t)\,dt + L \cdot \int_{\tilde{t}}^{\infty} u_{II}(t)\,dt. \qquad (60)$$

Inserting for u_I and u_{II} the expressions in Eqs. (54) and (57) and performing the integrations we obtain

$$K_L^o \Delta t_M = 3\,L^2 \ln\left(\frac{3L + u_i t}{3L}\right) + \frac{L\tilde{u}}{a} \qquad (61)$$

where $a = (10\nu/3L^2)$. The second term on the right turns out, under the substitutions below, to be relatively small.

Inserting typical values for the stratosphere of $\nu = 1.64 \times 10^{-4}\ m^2/sec$, $S = N(R_i)^{-1/2}$, $N = 0.225\,S^{-1}$, $L = 100$ m, and $R_i = 0.15$ in Eqs. (46), (58), (59), and (61) we arrive at

$$K_L^o\ \Delta t_M = 3.94 \times 10^5\ m^2 \qquad (62)$$

using

$$K_L^o = L\,u_i \qquad (63)$$

we find $K_L^o = 309\ m^2/sec$, hence

$$\Delta t_M = 1.28 \times 10^3\ sec \qquad (64)$$

which agrees* with $\Delta t_f = 1500$ seconds. Note, in passing, that K_L^o here far surpasses the limit of Eq. (41).

Of course, P* is related to Δt_M and Δt_f as we have seen; however, there is a subtle point to be cleared up regarding the definition of P*. In the calculations

*It should be mentioned that, had it turned out that $\Delta t_M >> \Delta t_f$, the assumption that we ignore transport between the (potentially) turbulent layers could have been violated. Once rendered turbulent, they could have churned for a very long time and eventually overlapped.

associated with Eq. (16) (Ref. 1) we were working with vertical profiles of unstable layers (potentially turbulent). In contrast, the P*, which occurs in the calculations by Lilly used in Eq. (1), refers to the probability of encountering turbulence in the horizontal direction. We, therefore, will put this distinction between vertical and horizontal determination of P* in evidence by using P_V* and P_H* respectively. The two cases of K_e are thus rewritten as

$$K_e = P_H^* K_L \frac{\Delta t_M}{\Delta t_f} \tag{35}$$

where the constant has been set equal to 1 in Eq. (36), and

$$K_e = \frac{P_V^* \overline{L^2}}{2 \Delta t_f} . \tag{2}$$

We must re-examine our results in the light of this new distinction.

First let us re-examine the difference between Δt_f and Δt_M. Δt_f is the duration of "potential" rather than actual turbulence because it represents the time between the event of the reduction of R_i to values below 0.25, and the event where R_i returns to values above 0.25 due to the effect of turbulent onset followed by layer expansion In other words, Δt_f is "growth to turbulence time." A source of error in the model is that sometimes R_i dips below 0.25 and then goes back to a value above 0.25 without an intervening billow event (see figures in Browning).[9] This is caused by variations in mean flow. If this were taken into account it would show that a certain fraction of potentially turbulent layers do not become turbulent and hence our estimate of K_e from Eq. (2) is an upper limit. One gets the impression from Browning's data that this artifact is not negligible; however, when one is more concerned about orders of magnitude rather than factors of 2 it appears perfectly safe to ignore this artifact. In contrast Δt_M is the "duration" of the turbulence as we have seen. As mentioned one of the uncertainties in the use of Eq. (35) is the value to use for K_L since at present one never knows Δt_M or K_L^O of any particular in situ measurement. The measured value of K_L may refer to a much decayed value for example. There are other uncertainties which will be discussed elsewhere, regarding the assumptions in going from raw data to K_L (Dewan).[18] The assumption of an inertial range, for example, is not valid, but the relation between K_L and ϵ seems, nevertheless, to have some validity (Panofsky et al).[19]

18. Dewan, E. M. (1979b) Anomalous -5/3 "Turbulence" Spectra in Stratified Fluids and Vertical Transport in Stratosphere I (unpublished, submitted to the J. Atm. Sci.).

19. Panofsky, H. A., and Heck, W. (1975) Stratospheric mixing estimates from heat flux measurements, The Natural Stratosphere of 1974, CIAP MONOGRAPH 1, DOT-TST-75-51 pp 6-90 to 6-92.

Next we examine the relations between P_H^* and P_V^*; Δt_M, Δt_f, and Δt_b. As we have seen

$$P_V^* = (\Delta t_f) / (\Delta t_{bP}) \tag{65}$$

where, now, we make the distinction that Δt_{bP} is the time at a given altitude, between occurrence of "potentially turbulent" events by introducing the second subscript, P. In the case of P_H^* we would have (assuming stationarity)

$$P_H^* = \frac{\Delta t_M}{\Delta t_{bT}} \tag{66}$$

where Δt_{bT} is the time between <u>actual</u> turbulent events. A derivation of Eqs. (65) and (66) would involve P^* as probabilities for which Δt_b is regarded as times between "hits" and Δt_f and Δt_M regarded as times between "trials" ("frames"). These times are the inverse of probability "frequencies."

Next we make the assumption (in the context of previously mentioned cautionary remarks)

$$\Delta t_{bP} \approx \Delta t_{bT} \tag{67}$$

and we turn to the question of the significance of setting $\Delta t_M = \Delta t_f$ in the context of the new distinction between P_H^* and P_V^*. From Eq. (65) and Eq. (66) together with Eq. (67), it is now obvious that this equality between Δt_M and Δt_f is the same thing as

$$P_V^* = P_H^* . \tag{68}$$

Thus we are led to the question of whether this is in accord with available experimental data. In Ref. 1, $P_V^* \sim 4.79 \times 10^{-2}$. In contrast the P_H^* values listed by Lilly ranged from 2×10^{-2} over water to 5.2×10^{-2} over high mountains. The equality Eq. (68) is therefore not in conflict with observation; and, from this fact we have obtained some confirmation that the theoretical agreement between the values of Δt_M and Δt_f (shown above) is indeed valid.

3.6 A List of Experimental and Theoretical Questions of Interest Raised by the K_e Estimation Problem

This report has raised a number of questions and it would be of convenience if the most important of these were listed together in one place.

1. Δt_{bT} and Δt_{bP} are assumed equal, but how accurate is this equality? The first is the time between actual turbulent events at a given altitude and the second is the time between events where $R_i < 0.25$. It would seem that the most direct approach to this would involve simultaneous radar and radiosonde observations in the stratosphere.

2. What is a good value for Δt_f? This was obtained "quasi experimentally" in Ref. 1, but more observations of the type in No. 1 above could decrease the uncertainty of its value. Δt_f is perhaps the most uncertain parameter in the vertical stack model for K_e.

3. The value of $P_V{}^*$ (and the associated P (L)) is available for only one geographic locality at present. One needs a more diverse set of measurements so that one could estimate the global value for this quantity.

4. A better experimental estimate for Δt_M of $\int_0^\infty K_L\, dt$ is needed. In situ balloon measurements located within turbulent layers would be helpful in this regard.

5. Since measurements of $P_V{}^*$, and $P_H{}^*$ and P (L) and so on, to date have been made only under ideal weather conditions, it would be useful to know how the estimates are altered by other weather conditions such as storms.

6. Some measurements of turbulence may have actually been measurements of gravity waves. Stewart (1969) has suggested a way to distinguish between the two phenomena and this should be investigated. Gravity waves cause no mixing, thus if they are mistaken for turbulence, an overestimate for K_e would result. (cf. Dewan,[20] in preparation).

7. All estimates of K_e due to local turbulence to date have assumed that turbulent layers form at random altitudes and random times. This assumption is so crucial that, if it were significantly violated, our model for K_e would have to be modified.* Crane[21] has given some radar data that could be interpreted as evidence that the assumption is in fact violated; and a theory by Dewan[20] in preparation involving trapped gravity waves would, if supported, imply that the assumption in question is indeed not valid. This should be experimentally investigated (perhaps best by radar means), and I consider it to be the most important unanswered question concerning stratospheric turbulence.

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20. Dewan, E.M. (1979a) Stratospheric Spectra Resembling Turbulence (unpublished, submitted to Science).

21. Crane, R.K. (1977) Stratospheric Turbulence Analysis (Air Force Geophysics Lab.) final report, ~~AFGL-TR-77-0207, ADO 47740.~~

*The modification would be accomplished by a change in the definitions Δt_f and K_L as well as their values.

Finally, there is the main question raised by this report, namely, is K_L large or small. In other words, which of the following relations is the valid one for the estimation of K_e,

$$K_e = \frac{\overline{L^2}\, P_V^*}{2\Delta t_f} \tag{69}$$

or

$$K_e = K_L\, P^* \tag{1}$$

If the previously mentioned experimental evidence is substantiated by direct measurements, then Eq. (1) would be ruled out.

4. CONCLUSION

The two methods to estimate K_e from small scale stratospheric turbulence were shown to be two extreme cases of the result of a vertical stack model. In one case K_e depends on turbulent layer thickness and Δt_f which is the time between profiles. In the other case these geometrical properties are replaced by a need to know K_L, the eddy diffusion within a layer.

It is not yet known which of the two estimates is correct. To determine that, one must have further knowledge of K_L and Δt_M defined in the text. These could be found by <u>in situ</u> measurements. Finally, a careful examination of the vertical stack model revealed a number of unanswered questions which were listed in order to expedite further experimental stratospheric research.

References

1. Junge, C. E. (1963) Air Chemistry and Radioactivity, Academic Press, New York, p 250.

2. Lilly, D., Waco, D., and Adelfang, S. (1975) Stratospheric mixing estimated from high-altitude turbulence measurements by using energy budget techniques. The Natural Stratosphere of 1974, CIAP MONOGRAPH 1, DOT-TST-75-51 pp 6-81 to 6-90.

3. Van Zandt, T. E., Green, J. L., Gage, K. S., and Clark, W. L. (1978) Vertical profiles of refractivity turbulence structure constant, Radio Sci. **13**:819-829.

4. Rosenberg, N. W., and Dewan, E. M. (1975) Stratosphere Turbulence and Vertical Effective Diffusion Coefficients, AFCRL-TR-75-0519, ERP No. 535.

5. Woods, J. D., and Wiley, R. L. (1972) Billow turbulence and ocean microstructure, Deep Sea Research and Oceanic Abst. **19**:87-121.

6. Woods, J. D. (1968) Wave-induced shear instability in the summer thermocline, J. Fluid Mech. **32**:791-800.

7. Phillips, O. M. (1969) The Dynamics of the Upper Ocean (Cambridge Univ. Press).

8. Koop, C. G. (1976) Instability and Turbulence in a Stratified Shear Layer, USCAE 134, Univ. of Southern California, School of Engineering, Dept. of Aerospace Engineering.

9. Browning, K. A. (1971) Structure of the atmosphere in the vicinity of large amplitude Kelvin-Helmholtz billows, Roy. Met. Soc. Quart. J. **97**:283-299.

10. Garrett, C., and Munk, W. (1972) Oceanic mixing by breaking internal waves, Deep Sea Research **19**:823-832.

11. Mantis, M. T., and Pepin, T. J. (1971) Vertical temperature structure of the free atmosphere at mesoscale, J. Geophys. Res. **76**:8621-8628.

12. Atlas, D., Metcalf, J., Richter, J., and Gossard, E. (1970) The birth of "CAT" and microscale turbulence, J. of the Atm. Sci. **27**:903-913.

13. Pasquill, F. (1962) Atmospheric Diffusion, Van Nostrand Co. Ltd.

14. Miller, R.W., Henry, R.M., and Rowe, M.G. (1965) Wind Velocity Profiles Measured by the Smoke-Trail Method at Wallops Island, Virginia (1959-1962) NASA TN D-2937, see also NASA TN D-4365 (1968).

15. Ingersoll, L.R., Zobel, O.J., and Ingersoll, A.C. (1954) Heat Conduction, University of Wisconsin Press (Madison, Wisc.).

16. Zimmerman, S.P., and Loving, N. (1975) Turbulent dissipation and diffusivities in the stratosphere using Richardson's technique. The Natural Stratosphere of 1974, CIAP MONOGRAPH 1, Final Report, DOT-DIAP, DOT-TST-75-51.

17. Tennekes, H., and Lumley, J.L. (1972) A First Course in Turbulence (The MIT Press, Cambridge, MA).

18. Dewan, E.M. (1979b) Anomalous -5/3 "Turbulence" Spectra in Stratified Fluids and Vertical Transport in Stratosphere I (unpublished, submitted to the J. Atm. Sci.).

19. Panofsky, H.A., and Heck, W. (1975) Stratospheric mixing estimates from heat flux measurements, The Natural Stratosphere of 1974, CIAP MONOGRAPH 1, DOT-TST-75-51 pp 6-90 to 6-92.

20. Dewan, E.M. (1979a) Stratospheric Spectra Resembling Turbulence (unpublished, submitted to Science).

21. Crane, R.K. (1977) Stratospheric Turbulence Analysis (Air Force Geophysics Lab.) final report, AFGL-TR-77-0207, ADO 47740.